Collaborative Inquiry for Learning Mathematics for Teaching (CIL-M)

A Professional Learning Framework for Studying Mathematics for Teaching

Session #3B – February 10, 2009
Division, Data Management, and Public Research Lesson

DSBN and NCDSB
The Literacy and Numeracy Secretariat
Who's Here?

Lincoln Centennial PS  
Pelham Centre PS  
St. Gabriel Lalemant CES

Smith PS  
St. Mary CES  
St. Denis CES
# Getting Started

Starting the feedback … “I implemented, I came expecting …”

<table>
<thead>
<tr>
<th>Region – Check one only:</th>
<th>Participant Responsibilities:</th>
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<tbody>
<tr>
<td>Barrie □</td>
<td>classroom teacher □</td>
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<tr>
<td>London □</td>
<td>SPED teacher □</td>
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<tr>
<td>North-West □</td>
<td>math coach □</td>
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<td>North East □</td>
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<td>Ottawa □</td>
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Strategies from our study group I implemented in my classroom ...  
How often?

<table>
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<th>I came expecting ...</th>
<th>I got ...</th>
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<table>
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<tr>
<th>I value ...</th>
<th>I still need ...</th>
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Two ideas/strategies I will implement ...

<p>| | |</p>
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BEFORE (Activation of Knowledge and Experience)
• Observations and Queries
• Teacher moderation from teacher’s classroom (includes discussion of one set of solutions (bansho))

DURING (Working On It)
• Do math to study
• Co-teaching public research lesson preparation (solve math for each lesson)
• Preparation for co-Teaching Public Research Lesson (discussing the lesson plan and anticipating range of solutions and relationships between the solutions)
• Co-Teaching Research Lesson

AFTER (Consolidation, highlights summary, practice)
• Teacher Moderation from public research lesson
• Discussion of public research lesson in terms of teaching through problem solving and co-teaching
• Highlighting classroom and school implementation strategies

Dates, Times, Location
1 - Dec 2 - St. Denis CES
2 - Jan 20 - Lincoln Centennial
3 - Feb 10 - St. Denis CES
4 - March 10 - Lincoln Cent
5 - March 31 - St. Denis CES
6 - April 28 - Lincoln Cent

All sessions are from 9:00 am – 3:30pm
Collaborative Inquiry for Learning Mathematics

CIL-M Aims

- To improve student learning and achievement in mathematics
- To improve instruction in mathematics
- To develop and support professional learning networks among teachers, principals, vice-principals, and superintendent across DSB Niagara and Niagara Catholic DSB
CIL-M Inquiry Learning Goals

- Understanding and applying teaching strategies that address student learning issues:
  - to provide a complete explanation and/or a justification using mathematical terms
  - to use mathematical language orally and in written forms to explain mathematical processes and
  - different ways to solve multi-step problems
  - appropriate problem solving strategies

- Studying **mathematics content and instruction**

- Developing, **planning, implementing mathematics lesson** and **assessing student learning** in classrooms within numeracy learning blocks, using evidence-based teaching strategies

- Using a **teacher moderation (assessment for learning)** process
Our Study Learning Goals
What’s Similar Among our School Goals?

• improving mathematical communication
• developing and implementing assessment for learning strategies
• implementing strategies for teaching and learning mathematics through problem solving
### Framework for CIL-M

**Beginning Roles and Responsibilities**

- **classroom and SPED teachers**
  - Implement classroom strategies, gather and analyse evidence of student learning, co-teach, provide feedback for CIL-M planning, record reflections

- **principals and vice principals**
  - Implement school strategies, monitor and analyse evidence of student learning, co-teach, provide feedback to CIL-M planning, record reflections

- **Lead teachers, math consultant, curriculum coordinator, principal or vice-principal, superintendent, School Effectiveness Lead, LNS student achievement officers,**
  - Co-plan, prepare, and/or co-facilitate CIL-M sessions, co-teach with teachers and principals/vice-principals in-between sessions
# Agenda - Session #3B - February 10, 2009

## MORNING:
- **9:00 - 9:45** Observations and Queries
- **9:45 - 10:30** Teacher Moderation from Classroom (Co-Teaching)
- **10:30 - 10:45** BREAK
- **10:45 - 11:00** Teacher Math Task and Discussion
- **11:00 - 11:45** Doing Mathematics for Teaching (Problem Solving)
- **11:45 - 12:00** Preparing for Co-Teaching Lessons (Do Lesson problems)
- **12:00 - 12:40** LUNCH

## AFTERNOON:
- **12:40 - 1:10** Preparing for Public Research Lessons - Solutions
- **1:10 - 2:10** Co-Teaching in 3 Lead Classrooms (Gr 4, 5/6, 6)
- **2:10 - 2:25** BREAK
- **2:25 - 3:15** Teacher Moderation from Public Research Lesson
- **3:15 - 3:25** Highlighting Classroom and School Strategies
- **3:25 - 3:30** Complete feedback form
From Last Session ... I Still Need

- Focus on my students’ math and look for what they can do rather than what they cannot do
- More practice, more, practice and more practice co-teaching and teaching through problem solving
- Do I better job of the highlights summary
- Better understanding of the vision, structure, purpose and process of bansho
- To determine where explicit instruction fits in
- Learn to see the mathematics in student solutions; understand what I am looking for in student group work in problem solving
  - Get better at questioning
  - Difference between co-teaching and problem-solving
  - Ways to communicate students thoughts
  - To make math connections between students’ different solutions
- More strategies for building a math talk community in my classroom
- Time to go through resources to learn more about this stuff
- Encourage all staff to engage
- Recognize and accept that this math is going to get messy sometimes in order to grow professionally
- Learn about the continuum of math expectations to better plan problem-solving lessons
Square Dancing to Discuss Observations and Queries

- Student engagement is always high when they are solving problems
- How do I know they learned
- Assessment – how can we assess problem solving when they work on groups
- What about the struggling students
- In co-teaching nice to have teacher/researcher feedback about my students
- Sometimes surprised at what researchers tell me about my students
- How do I plan for problem solving lessons
- Holding back from telling children is hard
- Learning to articulate the math we see is something I am working on
Observations and Queries …

Strategies/Dilemmas for Implementing Co-Teaching

- Difference between co-teaching in public research lesson and co-teaching in classes back at school
- Our other teachers are curious and want to start learning
- What resources can we use to learn to teach this way?
- We need resources to examine and analyse when we are planning so we can see different strategies than the ones we were taught
- I get to see more about what my students understand but I don’t know how to evaluate it
- When none of the students are getting it what can I do?
- How do we know what solutions to pick and whether to do bansho or congress
Observations and Queries ...
Teacher Actions for Teaching through Problem Solving
Observations and Queries … Co-Teaching

What is the difference between co-teaching as:
- as job-embedded professional learning
- as preparation for designing a public research lesson
- within a public research lesson?
Observations and Queries … Co-Teaching

• What is the purpose of co-teaching?
• What is the difference between co-teaching as:
  – as job-embedded professional learning
  – as preparation for designing a public research lesson
  – within a public research lesson?

from the LNS webcast - “Coaching for Student Success in Mathematics” <www.curriculum.org>
What’s the Purpose of Co-Teaching?

• Bring back student assessment data to the classroom teacher;
• Another set of eyes to watch children’s mathematical thinking to inform instructional decision-making - to analyse the mathematical data and reflect
• Need to work on getting assessment data from all students - part of improving of improving co-teaching
• Shift on the go easier (to respond to emerging patterns of student thinking - real time way) -
• Shifting to new instructional practices ... support through co-teaching - synergy (have a co-teaching group that stays together - large enough for diversity and constant for coherence
• Bounce ideas off each other throughout the lesson
• Dancing together -> coteaching
Co-Teaching Process Overview
Questions to Ask and Think About

Getting Started ... Ask Questions
• What is (are) the learning goal(s) of this math lesson?
• What is the lesson problem?
• How are you introducing the lesson? Activating their prior knowledge?
• What mathematics are you expecting students to show as they solve the lesson problem?

Observing Students ... Think About Assessment for Learning
• What mathematics are students modeling, saying, drawing when working on the lesson problem?
• How does the mathematics students are showing relate to the math learning goal of the lesson?
• What mathematical dilemmas are students showing? Why?
• What does the students’ mathematical communication sound like? How could it be improved?
Co-Teaching Process Overview
Collaboratively Problem Solving the Lesson

Discussing Observations of Students Throughout the Lesson

- specific mathematical details that relate to the lesson learning goal
- range of mathematics shown in student solutions
- students’ mathematical dilemmas
- appropriateness of time allocated for parts of the lesson

Making Instructional Decisions Throughout the Lesson

- determining appropriateness of parts of lesson as it relates to emerging evidence of student learning
- adjusting time allocated for student solving of problem, intermittent class discussion, whole class discussion (e.g., picking up pace, providing more time for some students)
- choosing questions to provoke thinking, to confront students’ mathematical dilemmas
- choosing student solutions for class discussion (e.g., bansho) based on mathematics criteria of lesson learning goal
Teacher Moderation (Assessment for Learning) in Mathematics

from the LNS webcast - “Investigating High Yield Strategies for Improving Mathematics Instruction and Student Learning”
Teacher Moderation of Classroom Work
Assessment for Learning

Three key questions for analysis:

- What mathematics are evident in students’ communication (oral, written, modeled)?
- What mathematical language should we use to articulate the mathematics we see and hear from students? (e.g., mathematical actions, concepts, strategies, tools)
- What mathematical connections can be discerned between students’ different solutions?

Analyse student work by:

- identifying the mathematics evident in student solutions to a lesson problem
- discern the mathematical connections between the solutions,

Purpose - Plan next steps instruction

- during the lesson
- for the next day lesson
Teacher Moderation of Classroom Work
Assessment for Learning
BEFORE

- 5 to 10 minutes only
- Activating students’ mathematical knowledge and experience that is directly related to the mathematics in the lesson problem
- Use a smaller problem similar to previous lesson problem
- Use student work responses for class analysis and discussion to highlight key ideas and/or strategies
**DURING**

- 15 to 20 minutes
- Understand the problem, Make a Plan, Carry Out the Plan

Understand the problem - Ask, “What information from the problem will we use to make a plan to solve it? Explain.”

- Teacher records below the problem the information the students identify in a list.
- Students solve the problem in pairs or in small groups
AFTER (Consolidation)

- 20 to 25 minutes
- Teacher selects 2 or more solutions for class discussion and decides which solution to share first, second, third.
- Teacher organizes solutions to show math elaboration from one solution to the next, towards the lesson learning goal
- Students (Authors) explain and discuss their solutions with their friends
- Teacher mathematically annotates solutions to make mathematical ideas, strategies, and tools explicit to students for learning
AFTER (Highlights/Summary)

- Teacher revisits in the different solutions the key ideas, strategies, and models of representation that are related to the lesson learning goal.
- Teacher records key ideas, strategies, and models of representation separately, so the students see the explicit focus of learning from the lesson.
AFTER (Practice)

• Teachers chooses 2 or 3 problems, similar to the lesson problem for students to solve individually (or in pairs as a scaffold).
• Problems are different by number (choice, size), problem contexts, or variation of problem needing to be solved.
• Students are asked to use a strategy different from the one they used in the lesson to solve the practice problems.
**BEFORE**
Activation

*<problem or previous day solutions>*

*Record 2 solutions to Highlight math ideas*

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**DURING**

*<problem>*

- “What information from the problem will we use to make a plan to solve the problem? Explain.

*Record the info In a list*

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**AFTER**

- Consolidation

(focused on knitting ideas together from one solution to another towards the learning goal)

- **mathematical annotations** - either on the board or on the student solutions to make explicit mathematical ideas, strategies, and tools and to show relationships between the solutions

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**AFTER**

Highlights/Summary

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**AFTER**

Practice

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Labels (e.g., mathematical strategies)
Teacher Math Task

The idol judges are coming to Lincoln Centennial to host a special school-wide audition for Canadian Idol. There are 265 students at Lincoln Centennial. If 30 people audition in 1 hour, how long will the auditions last? Solve in 2 different ways.

b. As you walk around, you see these solutions. What is your response to the students in the class? (If needed, turn the paper over to record your response.)

![Image showing solutions]
Today’s Co-Teaching Public Research Lesson
Co-Teachers, Insider and Outsider Researchers

A. Representing Division (Gr 4/5)
• Gr4 - divide two-digit (Gr5 - divide three-digit) whole numbers by one-digit whole numbers, using a variety of tools (e.g., concrete materials, drawings)
• Representing the ways they solved the problem using numeric symbols with concrete materials and pictures

B. Representing Division in a Variety of Ways (Gr 6)
* Gr5 - divide three-digit whole numbers by one-digit whole numbers using a variety of tools

C. Graph It! (Gr 5)
* Gr5 - Collect and organize discrete primary data and display the data in charts, tables, and graphs (including broken line graphs) that have appropriate titles, labels, and scales
* Describe how a set of data is collected and explain whether the collection method is appropriate

Mark with
(Christine G co-teaching facilitator)

Jenn with Dave and Dan
(MLK co-teaching facilitator)

Paul with
(KKZ co-teaching facilitator)
Prepping for the lesson

<table>
<thead>
<tr>
<th>Learning Goal (Curriculum Expectations)</th>
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<tbody>
<tr>
<td>a. Divide single-digit whole numbers by one-digit whole numbers, using a variety of tools (e.g., concrete materials, drawings)</td>
</tr>
<tr>
<td>b. Divide three-digit whole numbers by one-digit whole numbers using a variety of tools</td>
</tr>
<tr>
<td>Represent the ways they solved the problem using numeric symbols</td>
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<td>a. Divide single-digit whole numbers by one-digit whole numbers, using a variety of tools</td>
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<td>b. Divide three-digit whole numbers by one-digit whole numbers using a variety of tools</td>
</tr>
<tr>
<td>Describe the meaning of the parts of the traditional algorithm and the partial quotients</td>
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**Lesson Components**

**Before (10-15 minutes)**

John has 93 hockey cards and he wants to share them with 4 friends. How many cards will each friend get?

- a. How does the solution (traditional division algorithm) work? Turn and talk.
- b. Use ELM base ten cut outs to discuss the use of the manipulatives to show the traditional division algorithm using manipulatives.
- c. Record their mathematical actions using a number sentence to communicate the meaning of the parts of the traditional algorithm and the partial quotients.

**During (20 minutes)**

Here are 1 more solutions to the hockey card problem:

- a. How does the solution work?
- b. Use the base ten blocks to show your thinking (kids can relate to their own situation if they remember. In small groups of 4—one solution). --> one solution for 2 groups.
- c. Chart paper with space around the solution and cut-out base ten blocks.

**After (Consolidation) — 20 minutes**

- Discussion of their analysis of the solutions, highlighting details of methods
- Discussion about the mathematical relationship between the solutions—number of groups, size of the groups

**After (Highlights and Summary) — 5 minutes**

- Explicit relationship between repeated addition/multiplication to the divisor, repeated subtraction from the dividend, and the partial quotient division (3 solutions)
- What does the partials of the division sentence represent:
  - number of groups, size of the groups
  - dividend, divisor, and quotient

**After (Practice) — 5-10 minutes**

| Jan has 87 hockey cards and she wants to share them with 5 friends. How many cards will each friend get? |
| Jan has 182 hockey cards and she wants to share them with 5 friends. How many cards will each friend get? |
| Jan has 182 hockey cards and she wants to share them with 5 friends. How many cards will each friend get? |

**Graphing**

- Graph titles, labels, and scales
- Process of data collection for gathering sufficient data values (sample size) in different categories for valid comparison

**Choose another data set from the class collection.**

- a. Support the data using an appropriate type of graph. Explain your choice of the graph.
- b. Explain how the data could have been collected and decide whether the method of collection affected the data interpretation.
Teacher Work Samples – prepping for the lesson
Teacher Work Samples – prepping for the lesson
Assessment for Learning

Grade 6:
Solve a problem involving division of whole numbers using a variety of tools and strategies.

Record two responses to the problem here.

Record responses that students make.

Decide which responses will be shared 1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> according to math criteria.
Assessment for Learning

Grade 4
- Represent fractions using concrete materials, words, and standard fractional notation and explain the meaning of the denominator as the number of the fractional parts of a whole or a set, and the numerator as the number of fractional parts being considered.

Grade 6
- Represent, compare, and order fractional amounts with unlike denominators, including proper and improper fractions and mixed numbers using a variety of tools.
Getting on our Wikispace

- www.wikispaces.com
Mr. Cicchino's Class is learning about division. The question they answered yesterday was: John has 93 hockey cards and he wants to share them with 4 friends. How many cards will each friend get?

Here is the solution the grade 4/5 class came up with.

Before

- 23 in the hundreds place
- 13 in the tens place
- 12 in the ones place

23 divided by 4 equals 5 with 3 left over.

- Carry down the 3 in the ones place and add to the 12 in the tens place.
- 15 in the tens place divided by 4 equals 3 with 3 left over.
- Carry down the 3 in the ones place and add to the 12 in the tens place.
- 15 in the tens place divided by 4 equals 3 with 3 left over.

3 single groups of 4 children adding 20 + 3 gives 23 total.

The answer would be 23 with 1 remainder.

In groups of 4: 93 divided by 4 equals 23 with 3 left over.

$4 \times 20 = 80$

The total is 80 groups.

$4 \times 3 = 12$

7 groups in total.

$80 + 12 = 92$

$3 \times 80 = 240$

$240 + 92 = 332$

The total is 332.
Student work samples – grade 5 Data
Sample Student Journal Entries

Sample size - the total number of people you surveyed.
Partial data - half the number of people you surveyed.

Sample size - it means how many people participated in the event.
Partial data - it means only parts of the people voted.

Sample size:
I think sample size means number of things people or a math term. Partial data is data that is only parts of it.

Sample size - the whole amount of a group of people.
Partial data - part of the total of people being surveyed.
Sample Student Journal Entries

Sample size - not exactly the whole survey (not all the people) so you have partial data.

Partial data - not the complete data only part of it is complete. The other part isn't completely total.

Sample size = not exactly the whole school or all of the kids to get all the data, you have to have all of the kids or all of the whole school. If you surveying the class and there are 13 kids there and 3 are away then you don't know all the data, you have partial of it.

Sample size means like there are 15 kids in a class and only 8 of them go to church then that's a sample size.

Partial data means like part of the total like 16 kids only 13 kids vote.

Sample size mean what the total of a class or something eles. Partial data mean a part of total. For sample size is like the total of something for example 35 or 23 class mate. That sample size.
Sample Student Journal Entries

Sample size: The number of people surveyed.
I think people's names, size, and birthday are all about them.

I think sample size is where you have to know how many people are in the class and you need all the information from most of the people in the class.

Partial Data
Partial data is like having a whole or a quarter of a whole.

I think sample size is getting information from different people with different opinions.
I think partial data is data that can be different sizes.

Partial Data
Partial data is part of the data it's not all of the data it is just a part of the data collected.

Sample Size
Sample size is the full amount of the data like all of 23 students but only 5 of them voted.
Co-Teaching Lesson Protocols for Co-Teachers

Co-teachers provide prompts, questions, and circulate individually and together, among the students and teacher-researchers. Co-teachers may NOT tell students what to do or think.

The co-teachers can:

- gather evidence of student learning through observation (see, hear)
- redirect students to interact and share ideas and questions with one another
- ask students questions to help clarify, probe their thinking
- give feedback to students about their mathematical thinking
Co-Teaching Lesson Protocols for Teacher-Researchers

Teacher-researchers (insiders, outsiders) may NOT tell students what to do or think.

The teacher-researchers may gather and record:

- **Insiders (#1)** - mathematical evidence of student learning through observation (see, hear), using an assessment tool, like the LNS Seating Plan tool.

- **Outsiders (#2)** - details of the co-teaching process, strategies used, and instructional decisions made in relation to student assessment for learning data and watch 2 groups of students.
Teacher Moderation
Assessment for Learning

Three key questions for analysis:
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Analyse student work by:
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Purpose - Plan next steps instruction
• during the lesson
• for the next day lesson
Highlights of our Study Session
Classroom and School Implementation Strategies

• Bansho (highlight and summary; sequence of the solutions; annotating the info from the problem (What information will WE use to make a plan to solve the problem?); CONCEPT - collective expression of the student learning in the classroom); classification of student responses is not the end result
• Make mathematical connections between different solutions;
• Identify the mathematics that is evident in the solutions
• Co-teaching;
What Preparations Support Co-Teaching?

- Research the *mathematics for teaching* (content, pedagogy, student thinking) for the teachers’ unit of study.
- Do the problems (with other teacher(s)) ahead of time to get a range of mathematical thinking.
- Think through the mathematics criteria for bansho, gallery walk, and/or math congress as it relates to the curriculum expectations/learning goal of lesson.
- Engage in co-teaching regularly to improve the precision of this job-embedded professional learning approach.
Mathematics for Teaching to Study

- Study whatever topics the lead teachers are teaching that day (maybe get some notice few days before on the wiki)
- Choose a topic to study that WE all will implement March or April (come with ideas session 3)
- Choose a topic we can study, like mental math strategies (that is in the curriculum, but often we don’t have time to develop) - developed through
Preparation for Our Next Study Meeting …
Study Meeting #4 - March 10, 2009

Implement -
- Try two strategies with your school colleagues
- Co-teach at least one time per teacher) and be prepared to share your experiences.

Watch/Read -
- “Coaching for Student Success” webcast
- LNS “What Works … Learning Blocks” monograph
- “Mathematics Learning Blocks” (will be on Wikispace)

Bring:
- Ontario Curriculum, Grades 1 to 8, Mathematics (Revised, 2005)
- 6 classroom student work samples from one lesson problem

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# Complete the Feedback

**Region – Check one only:**
- Barrie
- London
- North-West
- North East
- Ottawa
- Toronto

**Participant Responsibilities:**
- classroom teacher
- SPED teacher
- math coach
- consultant/coordinator
- principal/VP
- SO
- SAO

**Strategies from our study group I implemented in my classroom ...**

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Two ideas/strategies I will implement ...

How often?